Field: Agri-business

Tools for planning in agriculture – Linear programming approach

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Boosting Adult System Education In Agriculture - AGRI BASE

Erasmus+ K2 Action Strategic Partnership

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1. Introduction

The material in front of you offers a practical presentation of the usage of Linear programming in the process of planning in agriculture and agribusiness. Today, the process of planning is unthinkable without using some of the qualitative or quantitative techniques which are efficinelty used within the other fields as well.

The operational research consist of a group, primary quantitative techniques based on a mathematical programming which are efficiently used in the various management processes. The linear programming is among the most used techniques from that group.

For a long time, the linear programming is used for planning in the field of agriculture and agribusiness, yet, there is still a room for its expansion.

The purpose of this course is to practically approach the possibilities and ways of usage of the linera programming in the various fields of agriculture to a wider group of beneficiaries.

The materials is so structured that in its first part the theme is elaborated theoretically, while further, thorugh practical examples of application of crop production and livestock production, the reader is guided through the rest of the necessary steps.

For the linear programming problem solving the Solver is used which is an Add-in and is part of MS Excel.

1.1. Planning in agriculture (in brief)

Due to its complexity and its wider spectrum, there are many definitions for planning. One of them could be: The planning is a process of determination the goals and defining the steps towards their accomplishment. Also, there are many different classifications of the planning. Some of them take the time horizon as a criteria on which the planning is based, others take the space

within the entity on which the planning refers, while others take the general (details) of the goals, etc.

Authors are conformed to that the planning as a process if one of the key functions of the (agro) management. Illustratively, it is shown as following:

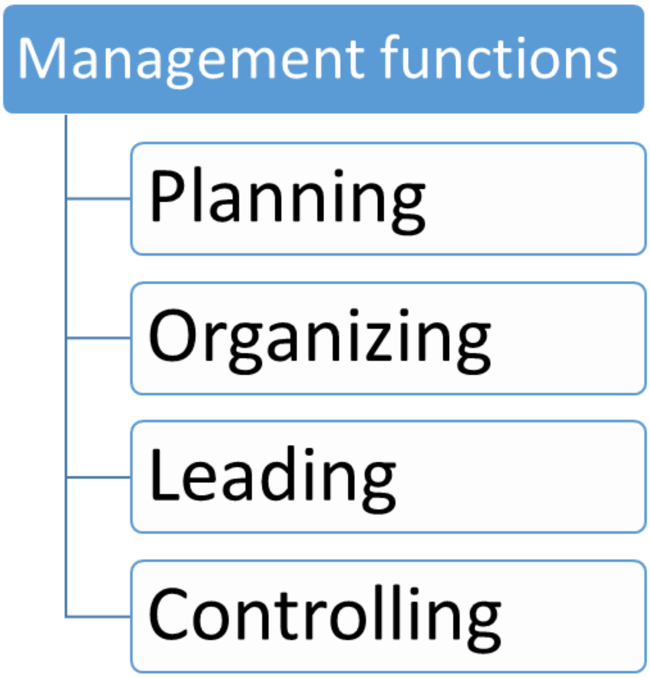


Figure 1. Management functions

As it is shown on the illustration, the planning is the first link of the management chain.

Even though the planning is a process directed towards future and is characterized with uncertaintly and risk, the mistakes done while planning are cost effective and not easily forgiven. It leads to the conclusion that the process of planning must be approached cautiously. The wrongly defined conclusion during planning lead to wrongly defined goals and priorities which lead in a wrong direction the entire organization.

Figure 2. Which direction is correct?

The scientific development in all fields stipulated the development of management and its functions, among them, the planning.

In time, the planning is becoming even more complex process with the usage of all the contemporary methods based on scientific findings from various scientific disciplines.

1.2. Some quantitative tools for agricultural planning

According to the complexity of the process of planning, there is a wide spectrum of quantitative and qualitative methods used in that process.

The operational research is a group of quantitavie methods used in the various management problems and they are frequent in the agromanagement problems.

By definition the „Operational research is a discipline that deals with the application of advanced analytical methods to help make better decisions.“ (Wikipedia).

Among the important characteristics of the Operational research are the following:

- Systematic approach
- Continuous research
- Optimization in solutions
- Complex research

According to the literature sources, the application of some methods of the Operational research is a phasal process, therefore, it is possible to define few successive phases which are common for many methods (Figure 3)

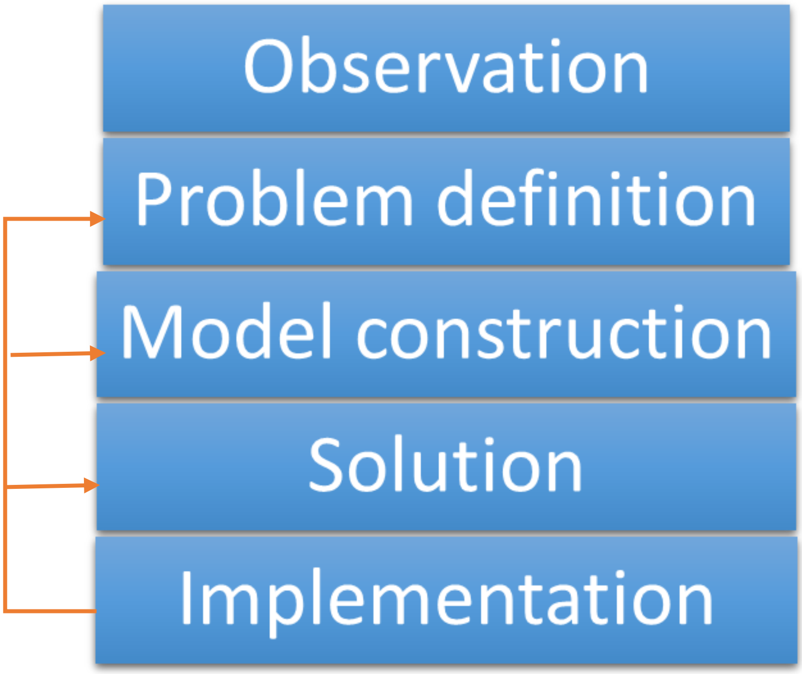


Figure 3. Phases of Operational research

There is a wide spectrum of quantitative methods in the operational research used in various fields of science and economy.

The mostly used methods in the field of agriculture and agribusiness are:

- Linear programming
- Non-linear programming - is the process of solving an optimization problem defined by a system of equalities and inequalities, collectively termed constraints, over a set of unknown real variables, along with an objective function to be maximized or minimized, where some of the constraints or the objective function are nonlinear.

- Dynamic programming - is a technique, which is used to analyze multistage decision process.
- Goal programming - is a branch of multiobjective optimization, which in turn is a branch of multi-criteria decision analysis (MCDA).
- Integer programming - is a technique, which ensures only integral values of variables in the problem.

This course is limited to the application of the linear programming as a tool in the process of planning in agriculture.

2. Linear programming approach

According to Wikipedia, Linear programming is „a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. Linear programming is a special case of mathematical programming".

Linear programs are problems that could be showen in matrix form as

Maximize $c^T x$

Subject to $Ax \leq B$

and $x \geq 0$

Where x represents the vector of variables (to be determined), c and b are vectors of (known) coefficients, A is a (known) matrix of coefficients, and $(\cdot)^T$ is the matrix transpose.

Linear programming approach encompasses four basic assumptions:

- Proportionality - The contribution to the objective function from each decision variable is proportional to the value of the decision variable.
- Additivity - The contribution to the objective function for any decision variable is independent of the values of the other decision variables.
- Divisibility - Each decision variable is allowed to assume fractional values.
- Certainty - Each parameter is known with certainty.

2.1. Linear programming models

The Linear programming is among the first and mostly used techniques of the Operational research in agriculture. There are fields of agriculture where the technique of linear programming can be successfully applied for solutions of various problems.

The models in linear programming differ upon many characteristics: the nature of problem to be solved, the size of the model, the level on which the planning refers, etc.

This technique can efficiently be applied on various levels in agriculture:

- Farm level
- Regional sector level
- National sector level.

There are many practical problems in agriculture which can efficiently be solved with the linear programming. Some of them are:

1. Least cost ration formulation,
2. Optimal resource allocation,
3. Otimisation structure of plant production,
4. Opimization of livestock production
5. Optimization of transport....

Regardless the problem, each model of linear programming consists of (Sallan et al. 2015):

$$
\begin{aligned}
\text{MAX } z &= c_1x_1 + c_2x_2 + \cdots + c_nx_n \\
\text{s. t. } & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1 \\
& a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2 \\
& \cdots \\
& a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m \\
& x_i \geq 0
\end{aligned}
$$

The model has the following elements:

- An objective function of the n decision variables xj . Decision variables are affected by the cost coefficients cj
- A set of m constraints, in which a linear combination of the variables affected by coefficients a_{ij} has to be less or equal than its right hand side value bi (constraints with signs greater or equal or equalities are also possible)
- The bounds of the decision variables. In this case, all decision variables have to be nonnegative.

Among the most considered criteria in Objective function in agriculture are:

- Maximize total production,
- Maximisation revenue/income,
- Minimisation cost,
- Minimize labour force use,
- Minimize fertilizer usage,
- Minimize water usage,
- Minimize total travelling distance of truck....

For the various problems which are solved with the linear programming, various constraints are defined. A sample for optimization problems constraint could be:

1. Avalilable arable agricultural land
2. Agrotechnical constratints (e.g. crop – rotation)
3. Available labour force
4. Available agricultural mechanization resource
5. Limitation of the barn/ stable capacity
6. Available capital
7. Martek constraints....

For example, we present one linear programming model for dairy farm which includes crop production (own forage crops production) (Vico et al, 2014).

Objective function:

$$(\max)\ f = \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij} x_{ij}$$

Constraints:

$$\sum_{i=1}^{p}\sum_{j=1}^{q} a_{ijkl} x_{ij} \leq = \geq u_{kl} \quad \mathrm{k} = 1,2,\ldots,\mathrm{r} \quad \mathrm{l} = 1,2,\ldots,\mathrm{s}$$

Conditions of non-negativity:

$$x_{ij} \geq 0, \mathrm{i} = 1,2,...,\mathrm{p} \quad \mathrm{j} = 1,2,...,\mathrm{q}$$

Indexes:

p – number of group activities

q – number of activities in a group

r – number of group limitations

s – number of limitations in a group

Activities:

x_{ij} ;i=1,2,...,p j=1,2,...,q

Constraints:

u_{kl} ;k=1,2,...,r l=1,2,...,s

Coefficients in a objective function:

c_{ij} ; i=1,2,...,p j=1,2,...,q

Coefficients in constratints:

a_{ijkl} - the quantity of j^{th}- activities within a i^{th}- group of activities l^{th}- constraints i r^{th}-group of limitations

Groups of activities:

- lines of farm production i=1 j=1
- fodder production at a farm i=2 j=1,2,...,8
- dry substance from the concentrated part of the meal i=3 j=1,2
- purchased fodder i=4 j=1,...,19
- other purchased inputs i=5 j=1,...,7
- other variable expenses i=6 j=1
- labor i=7 j=12
- finished products i=8 j=5

Groups of constraints:

- capacities k=1 l=1,2
- crop rotation k=2 l=1
- balance of nutritive k=3 l=1,...,13
- minimal l maximal components quantity in a meal k=4 l=1,...,12
- balance of other inputs k=5 l=1,...,8
- balance of labor k=6 l=1,...,12
- available labor k=7 l=1,...,12
- balance of mechanization k=8 l=1,...,4
- balance of finished products k=9 l=1,...,5

2.2. Steps in linear programming approach

The setting of the task in the linear programming incorporates a process of many successive steps. There is no a detail algorithm for all the problems, yet, the key steps in that process could be defined.

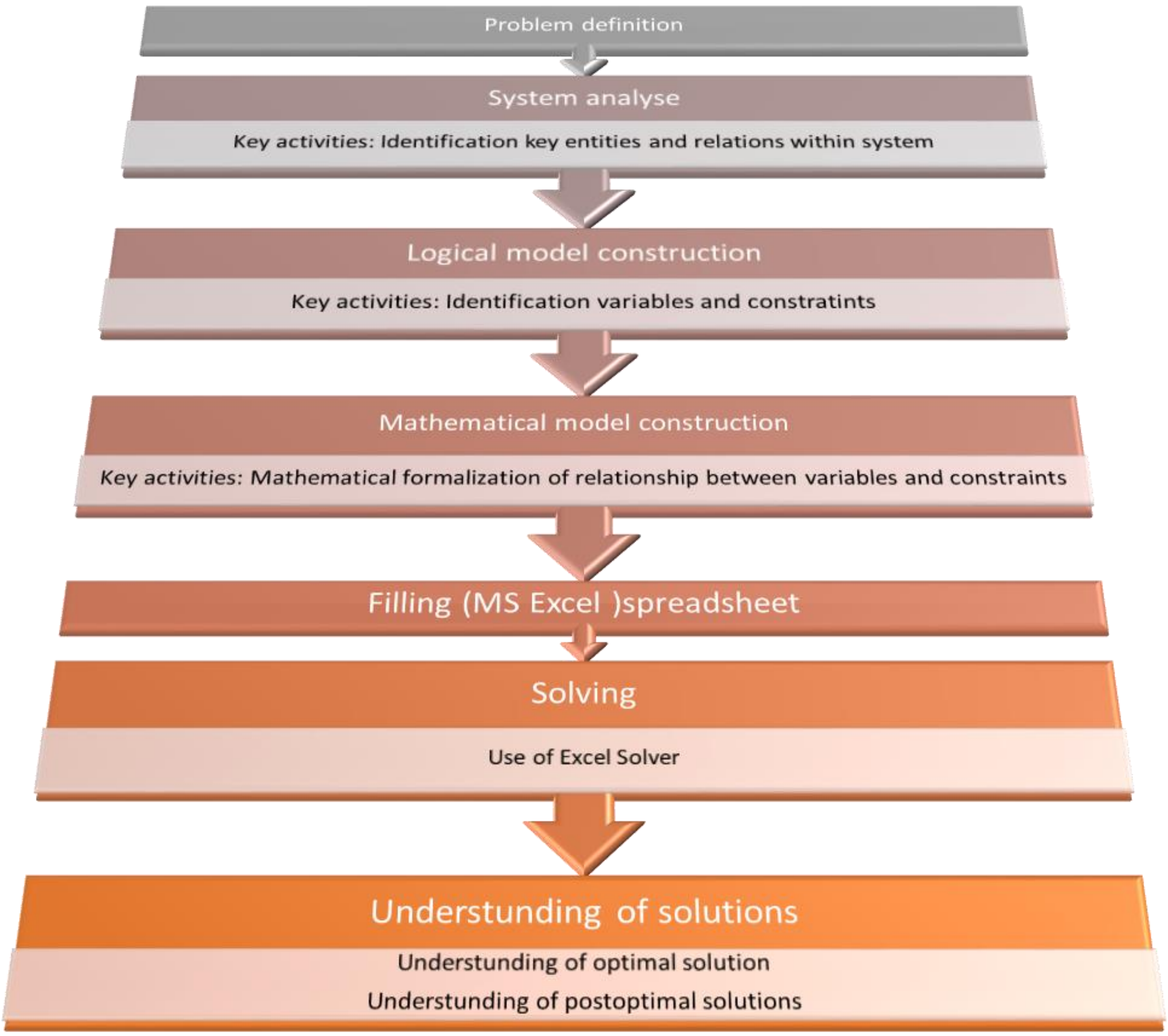


Figure 4. Steps for linear programming approach

3. Case studies

The linear programming is a very frequently used technique in solving the types of problems in the crop production and livestock production. The invention of optimal solution is what is common for them, and it is a maximum or a minimum objective function counting that all defined constraints are provided.

As our aim to to better introduce the reader with the subject topic and to clearly present the algorythms, in the following part of the material two Case studies will be presented:

1. Problem for defining the optimal structure in the crop production by using the maximization gross margin as a set criteria, and
2. Problema of optimization a ration in the livestock production by minimazing the least costs ration formulation as a set criteria.

3.1. Case study in crop production

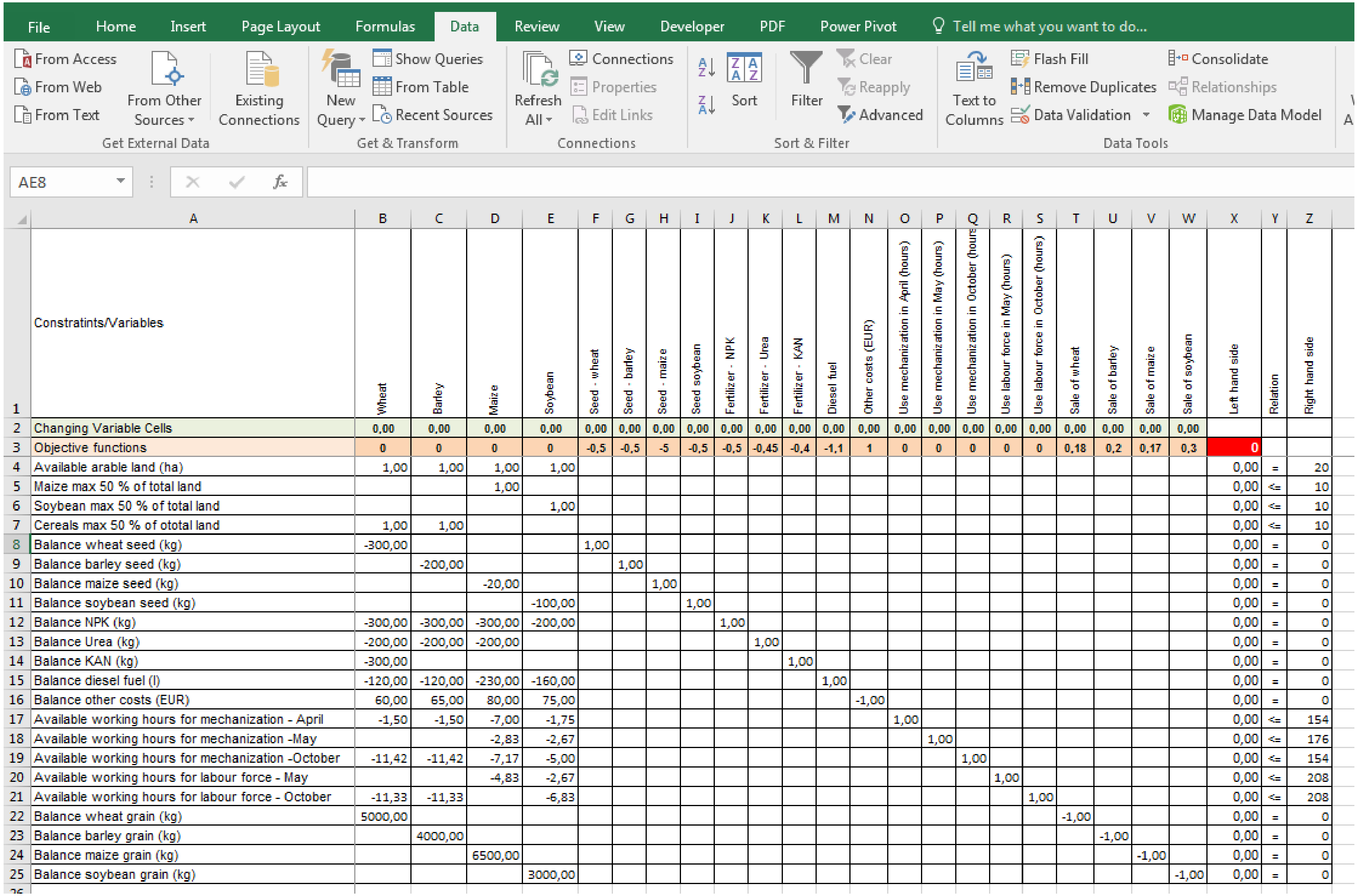
The linear programming is very often used for optimization of the crop production. The following part consists of a Case study on corp production.

The basic hypothesis in creating the Case study are:

- As a criteria for maximization in the Case study the Gross margin maximization is used
- Values are expressed in EURO
- Data is hypothetical and intended for didactical purposes
- A reduced model is created due to didactical purposes
- In disposal of the farm are 20 ha arable agricultural land
- There are possibilities for sowing four types of crops:
    - wheat
    - barley
    - maize
    - soya
- Maize could be sown on máximum 10 ha

- Cereals (wheat and barley) could be sown on maixmum 10 ha
- Soybean could be sown on maximum 10 ha
- Within the model the following inputs are shown: seedes, fertilizers, diesel fuel and other costs
- For each crop separately the mechanization working hours for the three months (April, May and October) are shown
- For each crop separately, the need of human labour is shown for the period of two moths (May and October)
- The available fond of mechanization working hours in April and October is 154, while in October is 176 working hours
- The available fond of human labour working hours in the given months is 208
- There is no rest of material input in the model (seed, diésel…..), which means that everything that is bought from the market will be spent

All data is inputed in an Excel Worksheet, as it is presented on the following Screenshot:



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Constraints/Variables	Wheat	Barley	Maize	Soybean	Seed - wheat	Seed - barley	Seed - maize	Seed soybean	Fertilizer - NPK	Fertilizer - Urea	Fertilizer - KAN	Diesel fuel	Other costs (EUR)	Use mechanization in April (hours)	Use mechanization in May (hours)	Use mechanization in October (hours	Use labour force in May (hours)	Use labour force in October (hours)	Sale of wheat	Sale of barley	Sale of maize	Sale of soybean	Left hand side	Relation	Right hand side
2	Changing Variable Cells	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00			
3	Objective functions	0	0	0	0	-0,5	-0,5	-5	-0,5	-0,5	-0,45	-0,4	-1,1	1	0	0	0	0	0	0,18	0,2	0,17	0,3	0		
4	Available arable land (ha)	1,00	1,00	1,00	1,00																			0,00	=	20
5	Maize max 50 % of total land			1,00																				0,00	<=	10
6	Soybean max 50 % of total land				1,00																			0,00	<=	10
7	Cereals max 50 % of ototal land	1,00	1,00																					0,00	<=	10
8	Balance wheat seed (kg)	-300,00				1,00																		0,00	=	0
9	Balance barley seed (kg)		-200,00				1,00																	0,00	=	0
10	Balance maize seed (kg)			-20,00				1,00																0,00	=	0
11	Balance soybean seed (kg)				-100,00				1,00															0,00	=	0
12	Balance NPK (kg)	-300,00	-300,00	-300,00	-200,00					1,00														0,00	=	0
13	Balance Urea (kg)	-200,00	-200,00	-200,00							1,00													0,00	=	0
14	Balance KAN (kg)	-300,00										1,00												0,00	=	0
15	Balance diesel fuel (l)	-120,00	-120,00	-230,00	-160,00								1,00											0,00	=	0
16	Balance other costs (EUR)	60,00	65,00	80,00	75,00									-1,00										0,00	=	0
17	Available working hours for mechanization - April	-1,50	-1,50	-7,00	-1,75										1,00									0,00	<=	154
18	Available working hours for mechanization -May			-2,83	-2,67											1,00								0,00	<=	176
19	Available working hours for mechanization -October	-11,42	-11,42	-7,17	-5,00												1,00							0,00	<=	154
20	Available working hours for labour force - May			-4,83	-2,67													1,00						0,00	<=	208
21	Available working hours for labour force - October	-11,33	-11,33		-6,83														1,00					0,00	<=	208
22	Balance wheat grain (kg)	5000,00																		-1,00				0,00	=	0
23	Balance barley grain (kg)		4000,00																		-1,00			0,00	=	0
24	Balance maize grain (kg)			6500,00																		-1,00		0,00	=	0
25	Balance soybean grain (kg)				3000,00																		-1,00	0,00	=	0

Photo 1. Screenshot – Defined Linear programming model in Excel spreadsheet

From the photo above it can be seen that the model consists of 22 variables and 22 constraints.

The labels of the variables are written in the first row. In the second row are the „Changing Variable Cells“and within the final calculation there will be given the values of each variable.

In the third row are the coefficiants in the Objective function. The values in this row depend on the set optimal criteria which is used in the given case. In this case it is the maximization Gross margin and in this row there are the separate prices for each variable which present the inputs paid on the market. Exp.: the price of 1 kg wheat seed is 0,5 EUR and the coefficient in cell F3 is -0,5.

The values are positive in the cell range T3:W3 and those areselling prices per kg produced agricultural products.

The contents in the cell span from A4:A25 are the labels of constraints.

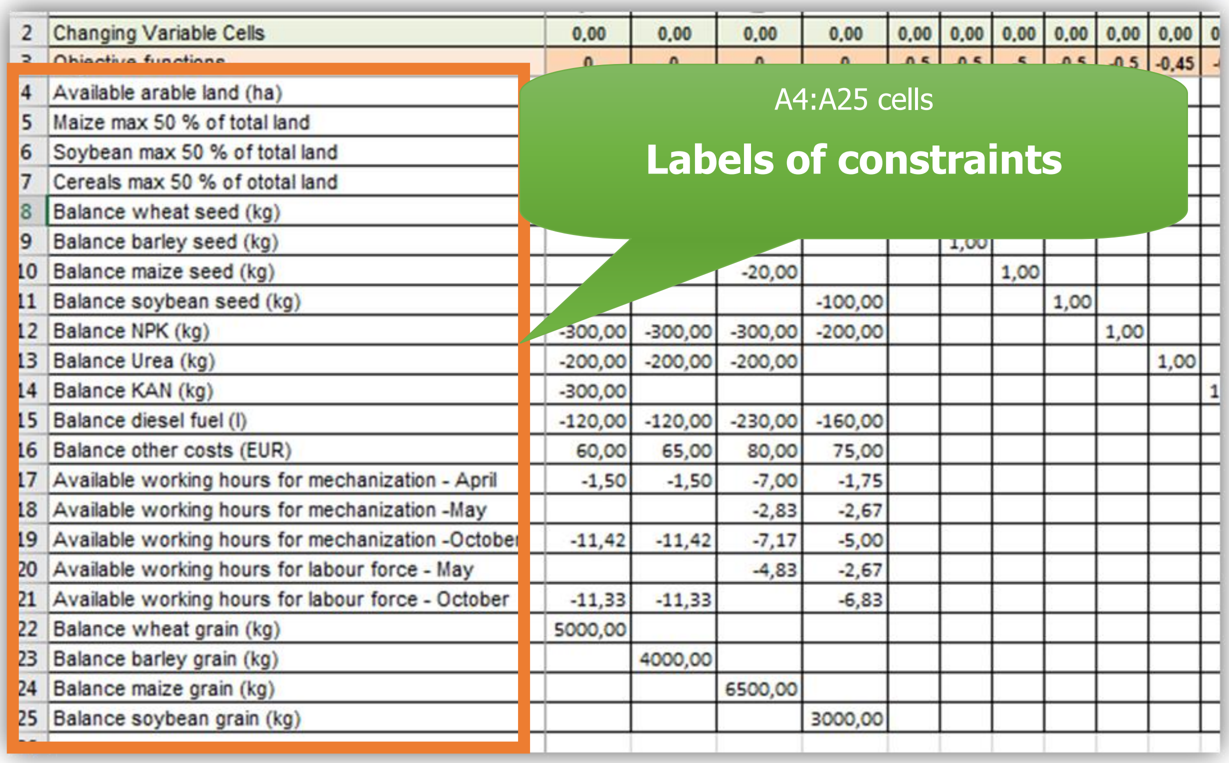


Photo 2. Screenshot – Constraints

The first four constraints refer on the constraints regarding the maximum usage of arable land.

The constraints in the span between the eighth and sixteenth row refer on the material inputs as well as the inputs given in currency (EUR).

The constraints in the span A17:A21 refer on the cost of mechanization and human labour.

The last four constraints refer on the sale of final products.

Cells in the span B4:W25 refer on the technical coefficients in the constraints matrix.

Photo 3. Screenshot – Technical coefficients

The coefficients are of different meaning. Their practical explanation is given on the example of wheat (the red frame)

- Number 1 in the fourth row shows that the wheat is comprised with constraint of the total arable land.
- The empty cells in the fifth and sixth row show that the constraints regarding maximal presence of maize and soybean do not refer on the wheat.
- Number 1 in the seventh row shows that the constraint of maximum presence of cereals up to 50% from the total arable land refers on the wheat.
- Number -300 in the eighth row shows that for 1 ha of wheat,300 kg seed wheat is needed.

- Next, for 1 ha wheat,300 kg NPK, 200 Urea, 300 KAN is needed (rows 12,13,14).
- For 1 ha wheat, 120 l diesel fuel and 60 EUR other costs are needed (rows 15 and 16).
- One ha of wheat correlates to 1,5 working hours of mechanization in April, 11,42 working hours in October and 11,33 working hours of human labour October (this refers only to the months included in the model).
- Row 22 shows that 5.000 kg grain is gained from 1 ha.

The coefficients for the other crops have the same meaning as for the wheat.

The next important part of the model is the Left hand side. Those are the cells in the span X4:X25. Within those cells the sums of the products technical coefficients are indicated in the cell span B4:W25 and the variables are given in the cell span B2:W2.

For this purpose the “installed” formula in Excel „SUMPRODUCT“is used.

The call upon the previous formula is given in the following Screenshot.

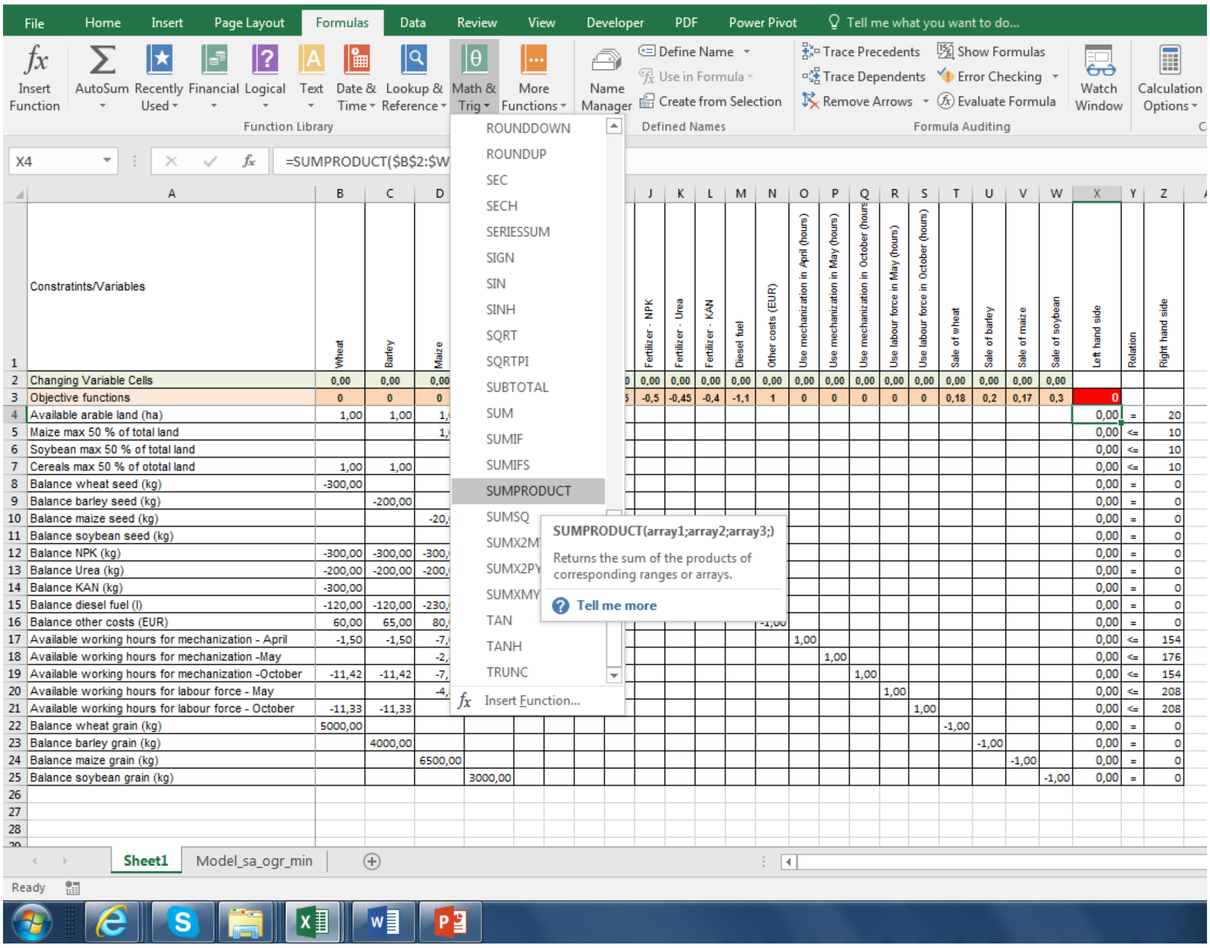


Photo 4. Screenshot - Launching SUMPRODUCT formula

SUMPRODUCT formula is used for getting the values in column X (Left hand side). Through this formula we multiply the technical coefficients found in the cell span B4:W25 with values of the variables given in the cell span B3:W3.

The syntax formula for row 12 (Balance NPK Constraint) is shown in the following screenshot.

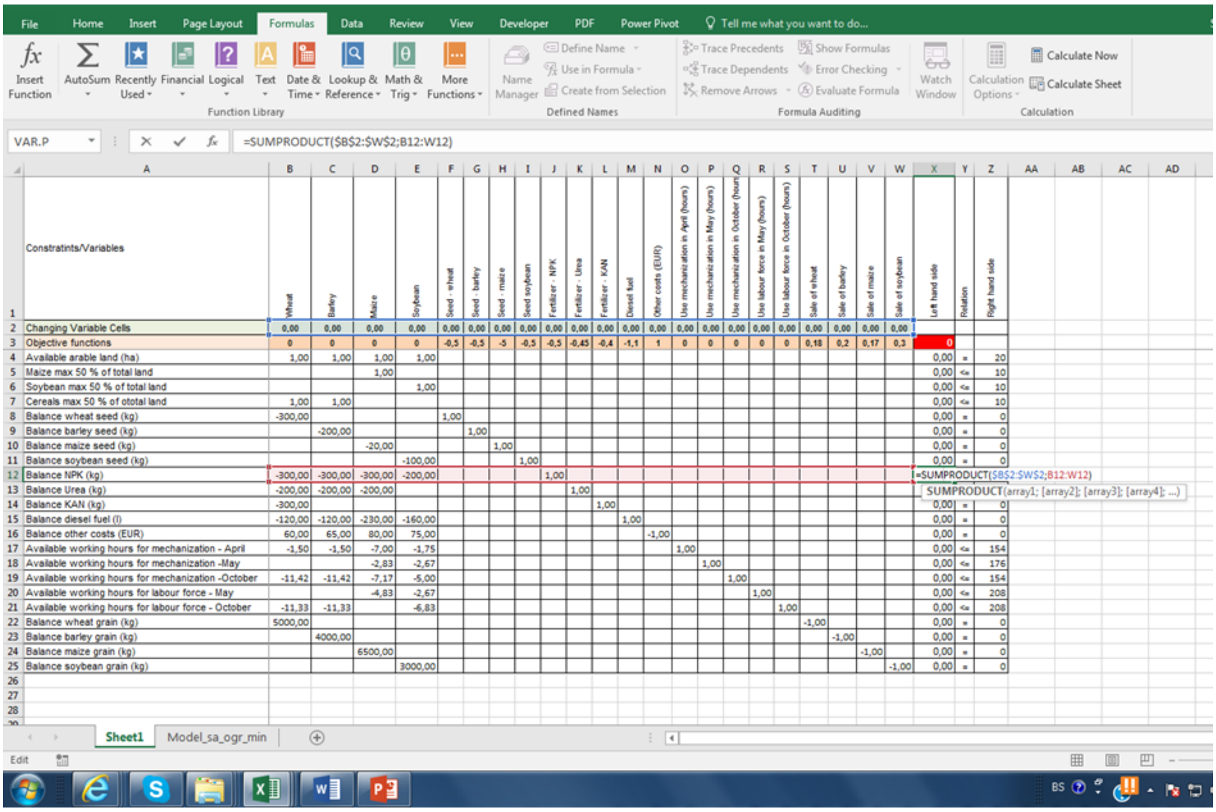
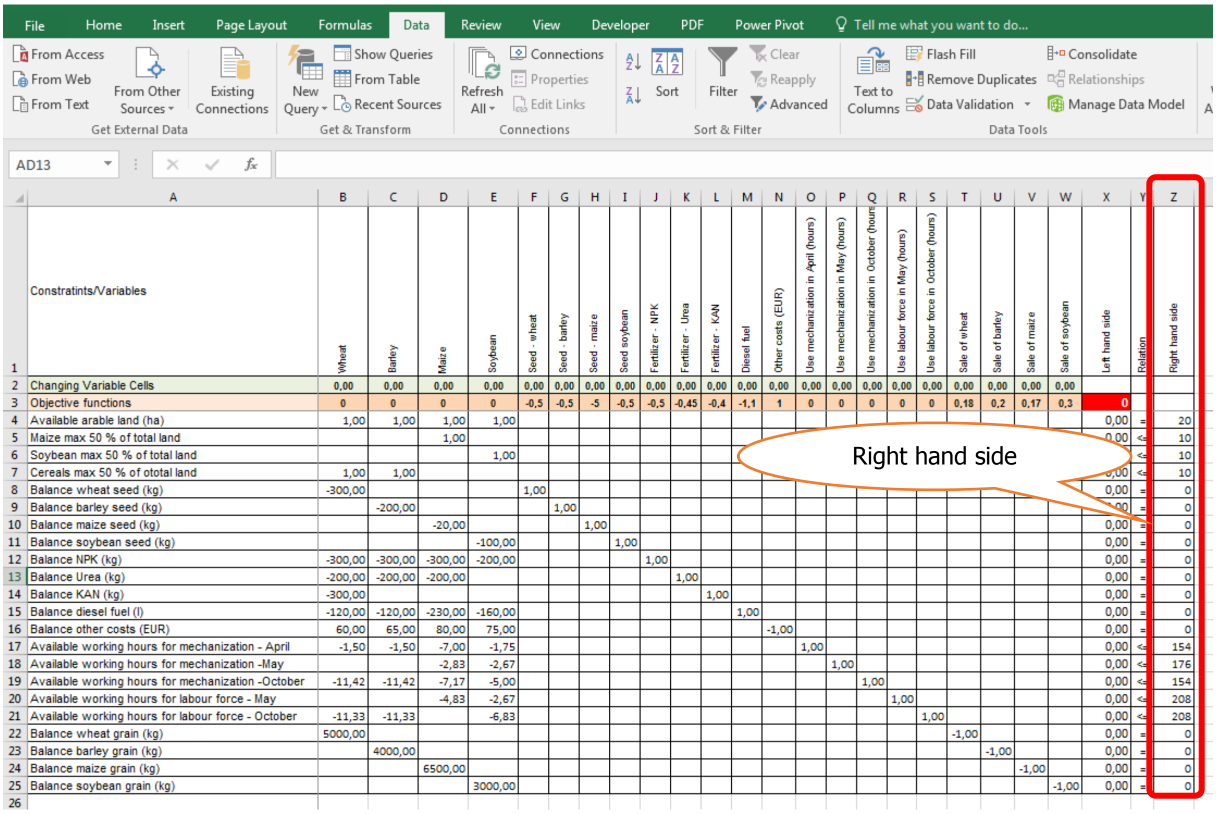


Photo 5. Screenshot – SUMPRODUCT Syntax

On the previous illustrations it is seen that in Y the relation signs are given. They don't have any mathematical meaning (it is later on marked in a Solver Dialog box), but here they are given due to didactical purposes.

In column Z the values representing the Right hand side are given. Here it is seend that on disposal of the farm there are 20 ha available arable land. It is also seen that the maize could be sown on maximum 10 ha, as well as the cereales and soybean. The available working hours for mechanization in Paril is 154 hours (row 17).

The position of Right hand side in the model as well as the values are shown in the following screenshot.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Constratints/Variables	Wheat	Barley	Maize	Soybean	Seed - wheat	Seed - barley	Seed - maize	Seed soybean	Fertilizer - NPK	Fertilizer - Urea	Fertilizer - KAN	Diesel fuel	Other costs (EUR)	Use mechanization in April (hours)	Use mechanization in May (hours)	Use mechanization in October (hours	Use labour force in May (hours)	Use labour force in October (hours)	Sale of wheat	Sale of barley	Sale of maize	Sale of soybean	Left hand side	Relation	Right hand side
2	Changing Variable Cells	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00			
3	Objective functions	0	0	0	0	-0,5	-0,5	-5	-0,5	-0,5	-0,45	-0,4	-1,1	1	0	0	0	0	0	0,18	0,2	0,17	0,3	0		
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6	Soybean max 50 % of total land				1,00																				<=	10
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10	Balance maize seed (kg)			-20,00				1,00																0,00	=	0
11	Balance soybean seed (kg)				-100,00				1,00															0,00	=	0
12	Balance NPK (kg)	-300,00	-300,00	-300,00	-200,00					1,00														0,00	=	0
13	Balance Urea (kg)	-200,00	-200,00	-200,00							1,00													0,00	=	0
14	Balance KAN (kg)	-300,00										1,00												0,00	=	0
15	Balance diesel fuel (l)	-120,00	-120,00	-230,00	-160,00								1,00											0,00	=	0
16	Balance other costs (EUR)	60,00	65,00	80,00	75,00									-1,00										0,00	=	0
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24	Balance maize grain (kg)			6500,00																		-1,00		0,00	=	0
25	Balance soybean grain (kg)				3000,00																		-1,00	0,00	=	0
26																										

Photo 6. Screenshot – Right hand side

Finally, the positioning of the cells into which the final result of the goal function will be given remains. In this case it is the cell X3. As same as in the case with the column Left hand side the value given in this cell is calculated with the formula SUMPRODUCT, as shown in the following screenshot.

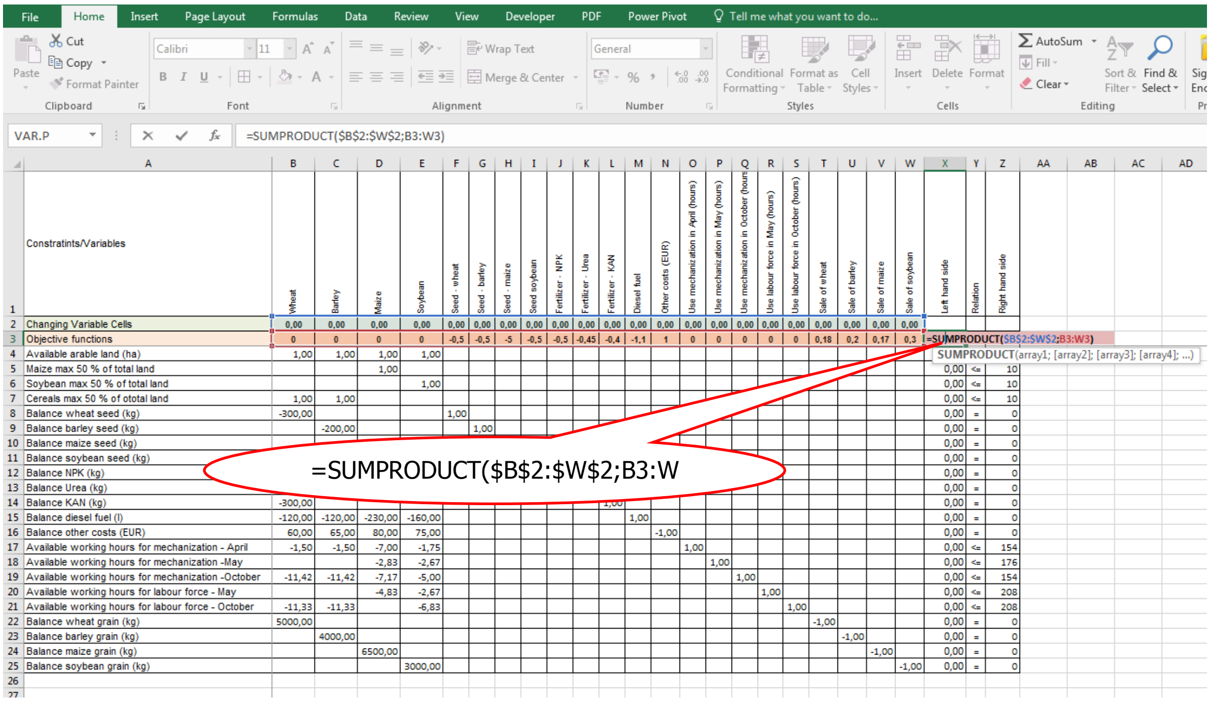


Photo 7. Screenshot – Target cell

The task set on this way in the MS Excel software is solved with the Solver. Solver is a Microsoft Excel add-in program you can use for what-if analysis. „Use Solver to find an optimal (maximum or minimum) value for a formula in one cell — called the objective cell — subject to constraints, or limits, on the values of other formula cells on a worksheet." (Excel 2016 Help).

The usage of Solvar requires its additional activating, and the way depends on the version of MS Excel.

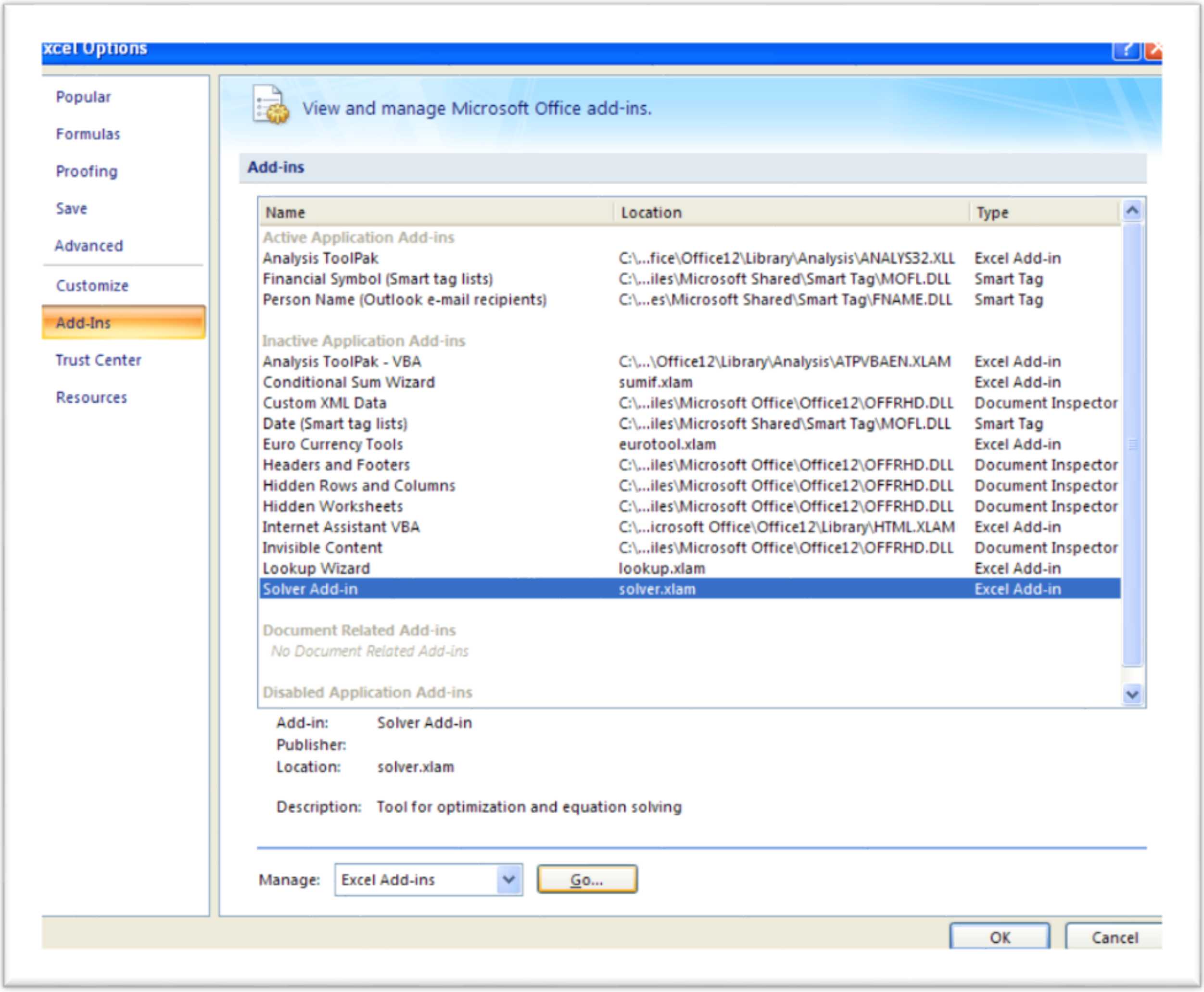


Photo 8. Screenshot – Solver activation

Following its installation, the Solver is approached thorugh these locations (MS Excel 2016) Ribbon tab name DATA > Command group ANALIZE. The visual illustration of Solver launching is shown in the following screenshot.

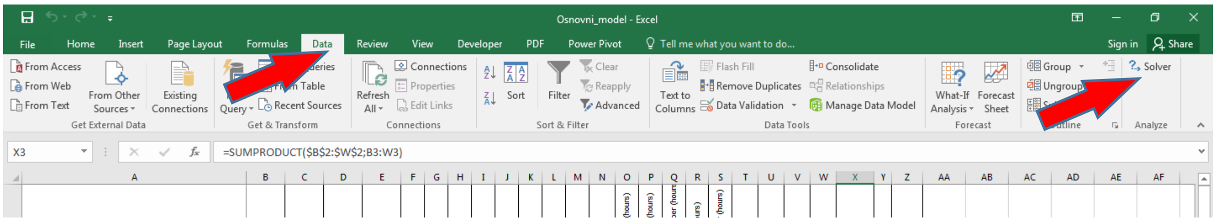


Photo 9. Screenshot – Solver launching

After launching the Solver a Dialog box Solver Parameters is opened into which all the necessary parameters for solving the set task of linear programming are inputed.

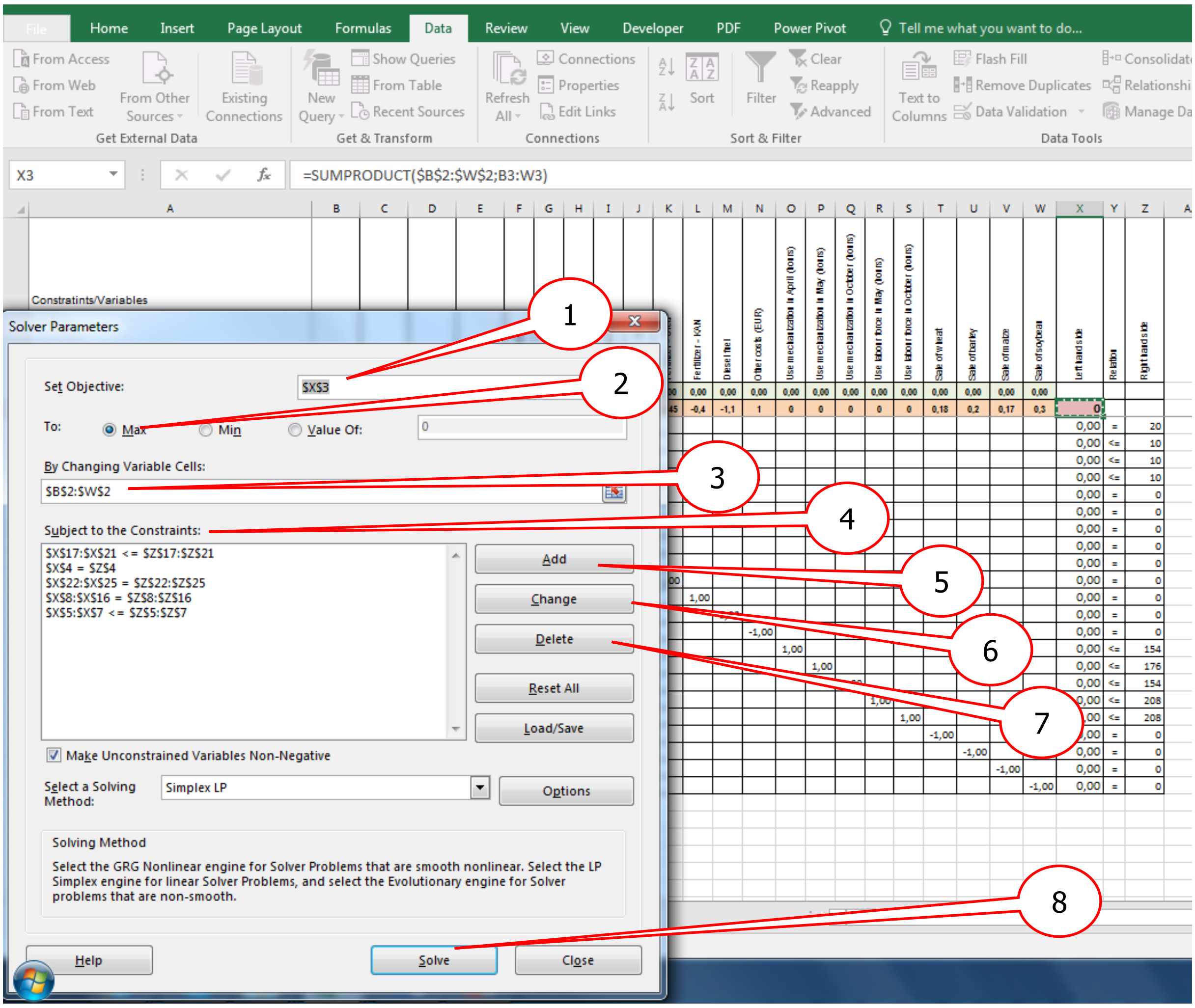


Photo 10. Screenshot – Solver Parameters

Explanation on the previous illustration:

1 – Objecstive cell –this field indicates the cell into which the final value of the objective function will be given. In this case it is cell X3.

2 – By click on „max"we present a will to maximazse the given criteria. In this case the criteria is Maximization Gross margin, therefore the button max is clicked.

3 – In the field „By Changing Variable Cells"the cell span is inputed into which the values of the variables will be given. In this case those are the cells in span B2:W2 and in the starting set of the task they are of 0 value.

4 – Field „Subject to the Constraints"– indicates the area where the inputed constraints are shown.

5 – Button for adding a constraint.

6 – Button for modifying a constraint.

7 – Button for deleting a constraint.

8 – Button for launching solving.

The adding of constraints is made by the button Add as it is shown in the following screenshot

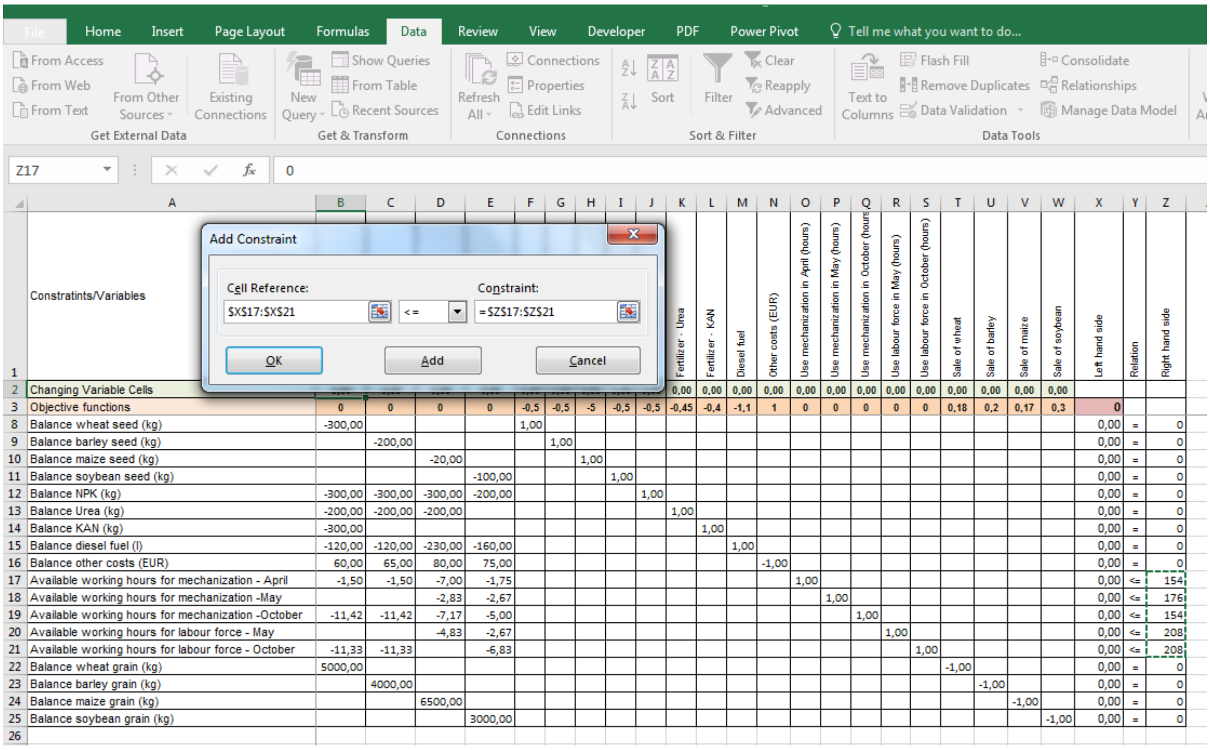


Photo 11. Screenshot – Adding constraints

The example with the previous illustrations refer on the constraints for the Available wokring hours for mechanization and Available working hours for labour force (rows from 17 to 21).

Solving

The objective of the linear programming problem solving is to get to a value for each included variable as well as for the value of the goal functions.

In the particular task, the values of variables should be written in the cell span B2:W2, while the value of the goal function should be indicated in the cell X3.

By clicking on the solve button in the Solver dialog box the solving of task by Solver is launching.

After the calculation, through the dialog box „Solver results“the Solver informs us that a solution is found. In this dijalog box we can choose some of the offered actions:

- Return to Solver Parameters Dialog
- Create post optimal reports
- Continue with showing solution

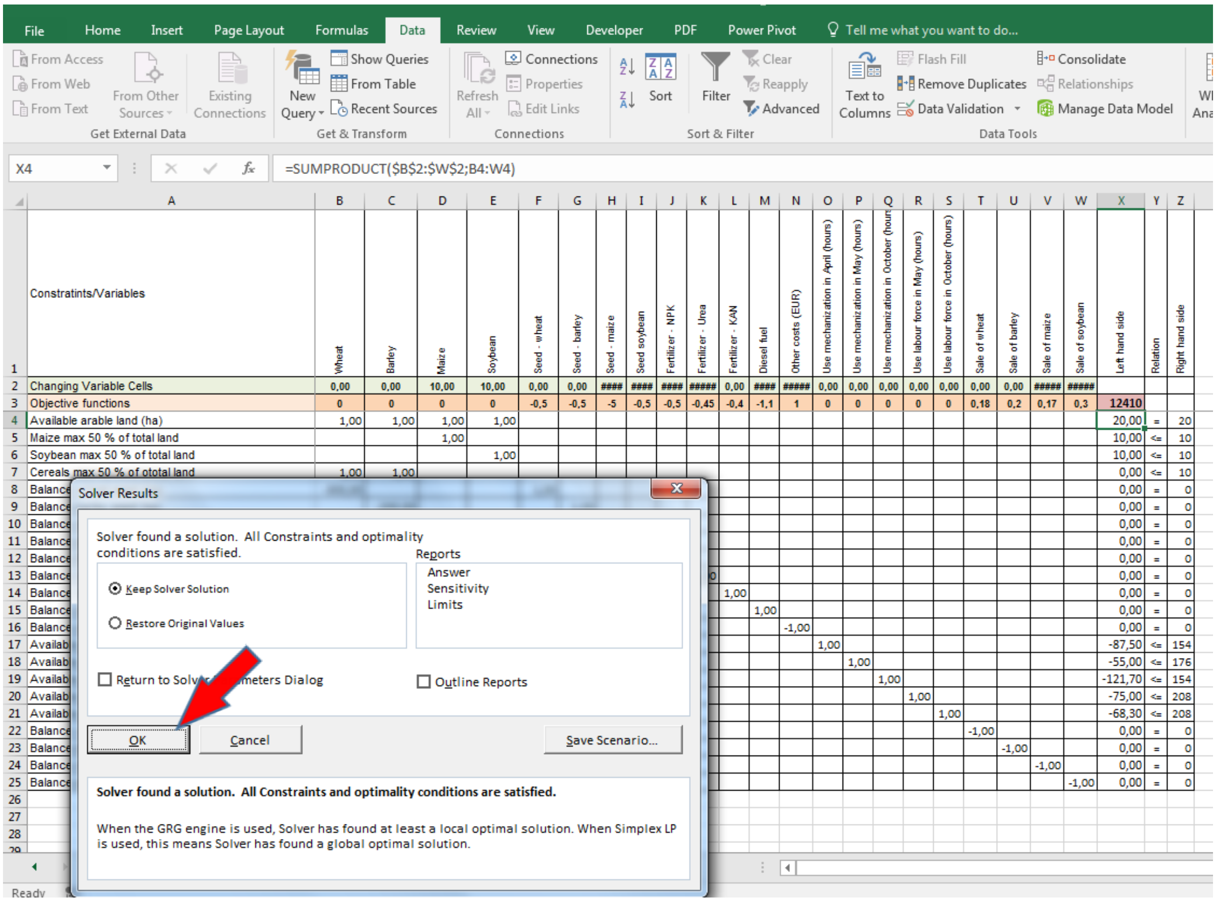
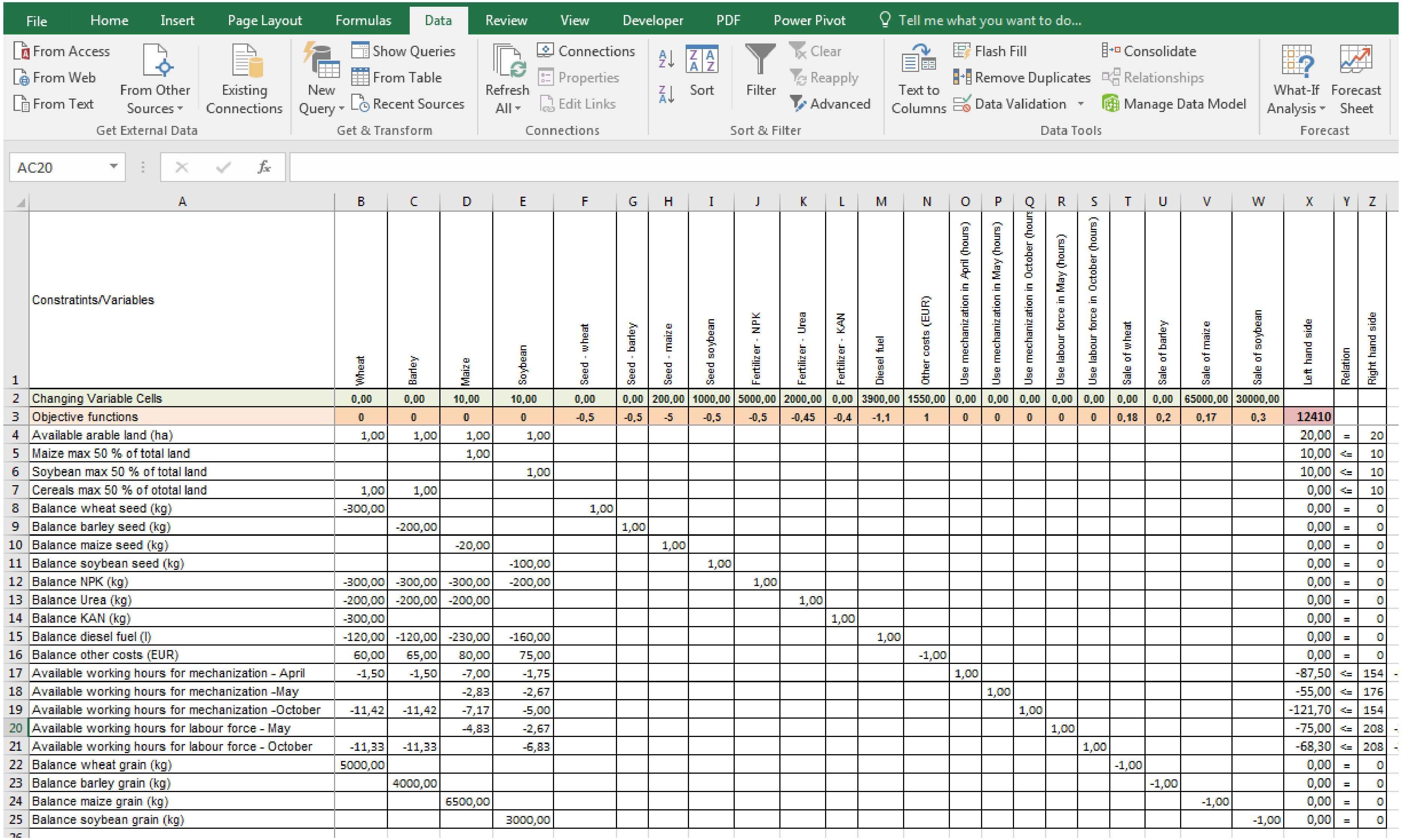


Photo 12. Screenshot – Adding constraints

After pressing OK, we can see the final solution wiith the requested values. The optimal structure of production consists of 10 ha maize and 10 ha soybean. By solving, all the constraints are provided, because neither the maize nor the soybean cross over 10 ha.

The total solution of the task is shown in the following screenshot.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	Constratints/Variables	Wheat	Barley	Maize	Soybean	Seed - wheat	Seed - barley	Seed - maize	Seed soybean	Fertilizer - NPK	Fertilizer - Urea	Fertilizer - KAN	Diesel fuel	Other costs (EUR)	Use mechanization in April (hours)	Use mechanization in May (hours)	Use mechanization in October (hours	Use labour force in May (hours)	Use labour force in October (hours)	Sale of wheat	Sale of barley	Sale of maize	Sale of soybean	Left hand side	Relation	Right hand side
2	Changing Variable Cells	0,00	0,00	10,00	10,00	0,00	0,00	200,00	1000,00	5000,00	2000,00	0,00	3900,00	1550,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	65000,00	30000,00			
3	Objective functions	0	0	0	0	-0,5	-0,5	-5	-0,5	-0,5	-0,45	-0,4	-1,1	1	0	0	0	0	0	0,18	0,2	0,17	0,3	12410		
4	Available arable land (ha)	1,00	1,00	1,00	1,00																			20,00	=	20
5	Maize max 50 % of total land			1,00																				10,00	<=	10
6	Soybean max 50 % of total land				1,00																			10,00	<=	10
7	Cereals max 50 % of ototal land	1,00	1,00																					0,00	<=	10
8	Balance wheat seed (kg)	-300,00				1,00																		0,00	=	0
9	Balance barley seed (kg)		-200,00				1,00																	0,00	=	0
10	Balance maize seed (kg)			-20,00				1,00																0,00	=	0
11	Balance soybean seed (kg)				-100,00				1,00															0,00	=	0
12	Balance NPK (kg)	-300,00	-300,00	-300,00	-200,00					1,00														0,00	=	0
13	Balance Urea (kg)	-200,00	-200,00	-200,00							1,00													0,00	=	0
14	Balance KAN (kg)	-300,00										1,00												0,00	=	0
15	Balance diesel fuel (l)	-120,00	-120,00	-230,00	-160,00								1,00											0,00	=	0
16	Balance other costs (EUR)	60,00	65,00	80,00	75,00									-1,00										0,00	=	0
17	Available working hours for mechanization - April	-1,50	-1,50	-7,00	-1,75										1,00									-87,50	<=	154
18	Available working hours for mechanization -May			-2,83	-2,67											1,00								-55,00	<=	176
19	Available working hours for mechanization -October	-11,42	-11,42	-7,17	-5,00												1,00							-121,70	<=	154
20	Available working hours for labour force - May			-4,83	-2,67													1,00						-75,00	<=	208
21	Available working hours for labour force - October	-11,33	-11,33		-6,83														1,00					-68,30	<=	208
22	Balance wheat grain (kg)	5000,00																		-1,00				0,00	=	0
23	Balance barley grain (kg)		4000,00																		-1,00			0,00	=	0
24	Balance maize grain (kg)			6500,00																		-1,00		0,00	=	0
25	Balance soybean grain (kg)				3000,00																		-1,00	0,00	=	0

Photo 13. Screenshot – Final values

On the previous screenshot in the second row, the final values of variables are visible. So it could be seen that for the proposed structured it was needed to spend 5.000 kg NPK, 2.000 kg Uree or 3.900 l diesel fuel.

Also, based on the final values it is seen that only the constraint „Available working hours for mechanization –October“ (Row 19) used in a somehow higher percent (121,70/157=79 %). The other constraints refereing on the mechanization and labour force are used in a slightly less percentage.

Upon this hypothetical example a conclusion can be made meaning that the farm has on its disposal more resources which are not adequately used. Those could be information inputs during the elaboration of the strategic farm plan.

The solving of the linear programming task offers even more useful data, in particulary if a posto-optimal analyses is applied within the task.

According to Vico et al. (2014) „Post optimum analysis could be divided into three functional units.

The first part is called the report of answers and it is related to the analysis of providing certain constraints.

The second part of the post optimum analysis refers to the sensitive analysis. By this analysis, information on the reliability of the optimal solution was obtained, because if the limits of the optimality of individual coefficients spread, the solution is more reliable and vice versa. Within the determination of each parameter in the objective function there are no changes of the optimal value, but only the value of the objective function increases. By sensitivity analysis, it is possible to determine the competitiveness of the variable in the objective function.

The third part of the post optimum analysis represents analysis of sensitivity of certain restrictions".

3.2. Case study in livestock production – Least cost ration formulation

The second practical example onto which the usage of linear programming is shown refers on the least cost ration formulation. In this example there will

be no detail explanation of the procedure of usage of MS Excel Solvera, since it was explained in the previous example.

The criteria which is used in ths task is the minimization of the ration costs. The task is created by using the existing literature data (Krstic et. Al, 2000) with some modificatons.

Task:

To make an optimal ration for cows during the summer period. Considering the average weight (500kg) and daily lactation of cows (15 kg 4% masti), the daily needs of important nutritives are:

Table 1. Daily needs for dairy cow

No.	Component	Request
1.	Energy units (E.U.)	12,5 kg
2.	Digest protein (D.G.)	1,45 kg
3.	Calcium (Ca)	0,09 kg
4.	Phosphorus (P)	0,06 kg
5.	Maximum green biomass	40 kg

Table 2. Chemical composition of nutrients

Nutritious		E.U.	D.P.	Ca	P	Price
Mark	Name	**(kg)**	**(kg)**	**(kg)**	**(kg)**	**(EUR/kg)**
X_1	**Maize meal**	**1,222**	**0,057**	**0,0004**	**0,0025**	**0,150**
X_2	**Sunflower pellets**	**0,901**	**0,336**	**0,0028**	**0,0008**	**0,300**
X_3	**Sobean pellets**	**1,190**	**0,387**	**0,0062**	**0,0058**	**0,350**
X_4	**Wheat meal**	**0,712**	**0,126**	**0,0012**	**0,0100**	**0,150**
X_5	**Fresh alfalfa**	**0,172**	**0,036**	**0,0035**	**0,0007**	**0,030**
X_6	**Fresh maize**	**0,202**	**0,015**	**0,0013**	**0,0003**	**0,025**
X_7	**Bone meal**	**0,000**	**0,000**	**0,3160**	**0,1460**	**0,450**

List of constraints:

Constraint for green fresh biomase:

I $X_5 + X_6 \leq 40$

Constraint for energy:

II $1{,}222X_1 + 0{,}901X_2 + 1{,}19X_3 + 0{,}712X_4 + 0{,}172X_2 + 0{,}202X_6 = 12{,}5$

Constraint for digest protein:

III $0{,}057X_1 + 0{,}336\ X_2 + 0{,}387X_3 + 0{,}126X_4 + 0{,}036X_5 + 0{,}015X_6 = 1{,}45$

Constraint for calcium:

IV $0{,}0004X_1 + 0{,}0028\ X_2 + 0{,}0062X_3 + 0{,}0012X_4 + 0{,}0035X_5 + 0{,}0013X_6 + 0{,}316X_7 = 0{,}09$

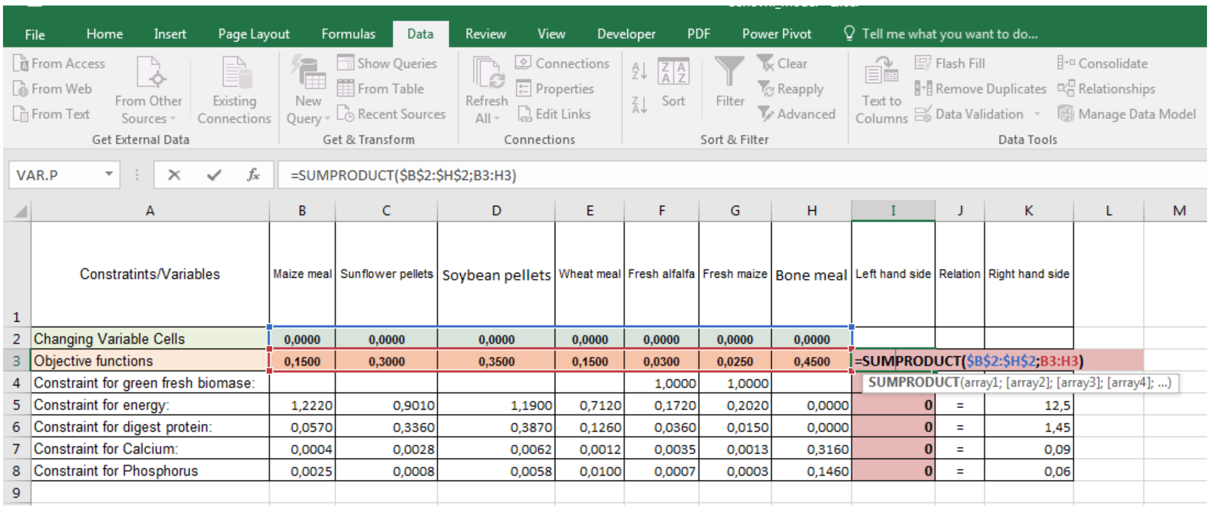
Constraint for phosphorus:

V $0{,}0025X_1 + 0{,}0008\ X_2 + 0{,}0058X_3 + 0{,}01X_4 + 0{,}0007X_5 + 0{,}0003X_6 + 0{,}146X_7 = 0{,}06$

Objective function:

$F = 3X_1 + 4X_2 + 6{,}5X_3 + 3X_4 + 0{,}3X_5 + 0{,}25X_6 + 4{,}5X_7 \rightarrow \min$

On the following screenshot a set model in MS Excel worksheet could be seen.



	A	B	C	D	E	F	G	H	I	J	K
1	Constratints/Variables	Maize meal	Sunflower pellets	Soybean pellets	Wheat meal	Fresh alfalfa	Fresh maize	Bone meal	Left hand side	Relation	Right hand side
2	Changing Variable Cells	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000			
3	Objective functions	0,1500	0,3000	0,3500	0,1500	0,0300	0,0250	0,4500	=SUMPRODUCT($B$2:$H$2;B3:H3)		
4	Constraint for green fresh biomase:					1,0000	1,0000		SUMPRODUCT(array1; [array2]; [array3]; [array4]; ...)		
5	Constraint for energy:	1,2220	0,9010	1,1900	0,7120	0,1720	0,2020	0,0000	0	=	12,5
6	Constraint for digest protein:	0,0570	0,3360	0,3870	0,1260	0,0360	0,0150	0,0000	0	=	1,45
7	Constraint for Calcium:	0,0004	0,0028	0,0062	0,0012	0,0035	0,0013	0,3160	0	=	0,09
8	Constraint for Phosphorus	0,0025	0,0008	0,0058	0,0100	0,0007	0,0003	0,1460	0	=	0,06

Photo 14. Screenshot – Least cost meal formulation model in Excel spreadsheet

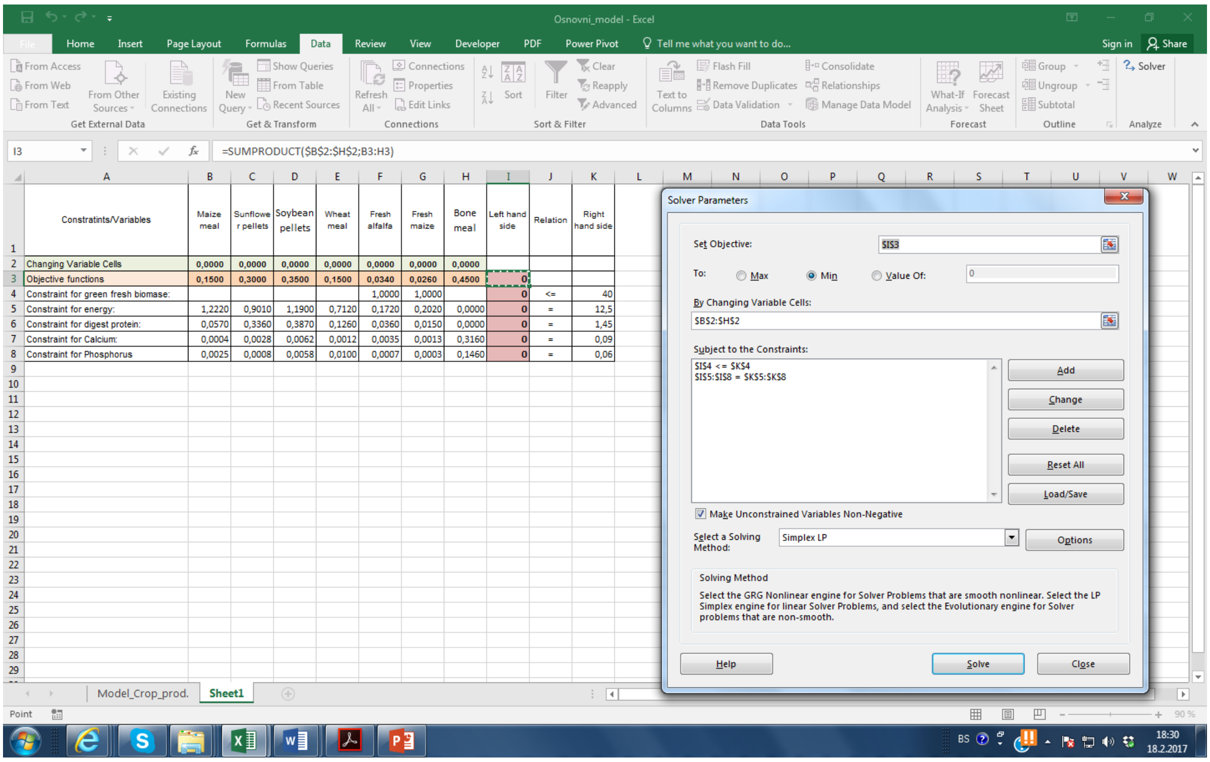


Photo 14. Screenshot – Filled Solver parameters dialog box

Solver came to the solution in the seventh iteration. In the final solution the structure of a daily ration for a dairy cow can be easily determined.

Table 3. Final values for ration formulation

Constraints/Variables	Maize meal	Sunflower pellets	Soybean pellets	Wheat meal	Fresh alfalfa	Fresh maize	Bone meal	Left hand side	Relation	Right hand side
Changing Variable Cells	**1,8156**	0,0000	0,0000	**3,6634**	**13,5675**	**26,4325**	**0,0096**			
Objective functions	0,1500	0,3000	0,3500	0,1500	0,0340	0,0260	0,4500	**1,97471**		
Constraint for green fresh biomass:					1,0000	1,0000		**40**	<=	40
Constraint for energy:	1,2220	0,9010	1,1900	0,7120	0,1720	0,2020	0,0000	**12,5**	=	12,5
Constraint for digest protein:	0,0570	0,3360	0,3870	0,1260	0,0360	0,0150	0,0000	**1,45**	=	1,45
Constraint for Calcium:	0,0004	0,0028	0,0062	0,0012	0,0035	0,0013	0,3160	**0,09**	=	0,09
Constraint for Phosphorus	0,0025	0,0008	0,0058	0,0100	0,0007	0,0003	0,1460	**0,06**	=	0,06

As it could be seen from the Table, for a daily ration it is needed:

Maize ration	1,82 kg
Sunflower pellets	0,00 kg
Soybean pellets	0,00 kg
Wheat ration	3,66 kg
Fresh alfalfa	13,57 kg
Fresh maize	26,43 kg
Bone ration	0,096 kg

By congregating the quantity of Fresh alfalfa (13,57 kg) and Fresh maize (26,43) it is visible that the constraint number 1 - Constraint for green fresh biomass is totally used. It means that the green fresh nutritient is more competitive than the rest of the nutritients.

The linear programming even offers data per levels of competitiveness of certain variables (in this case the nutritients). It is acquired by the postoptimal analyses, yet it os over the scope of this course.

4. Conclusion

The agribusiness is becoming a field into which various tools are applied which serve as a support in the realization of the various functions of management. The today's planning process in the agribusiness is unthinkable without the usage of some of the quantitative techniques.

The operational researches offer variour techniques which could efficiently be applied in the process of planning in agribusiness. The linear programming is one of the most frequently used quantitative techniques. There are many practical problems in the field of agribusiness which could be solved by linear programming.

The relatively good accessibility of software tools provides for the wider usage of the linear programing. The Add-In Solver within MS Excel is one of the mostly approached such tools.

This material presents practical examples for creating models and solution of problems for linear programming for crop production and livestock production.

5. References

Krstic, B., Tomic, R., Sorak, G. (2000): Organizacija I ekonomika stočarske proizvodnje. Praktikum. Službeni Glasnik. R. Srbija.

Sallan, J.M., Lordan, O., Fernandez, V. (2015): Modeling and solving linear programming with R. Universitat Politècnica de Catalunya. DOI: http://dx.doi.org/10.3926/oss.20

Vico, G., Rajic, Z., Peshevski, M., Sorajic, B. (2014): Sensitivity Of Input Prices In Cattle Production. PROCEEDINGS OF PAPERS. The Role of Agricultural Economic Sciences in Agriculture and Rural Development in the Balkan Countries. AAEM 9th International Conference, 24-25 April 2014, Ohrid, Macedonia.

Vico, G. (2012): Optimization of cattle production based on multiple criteria in the Republic of Srpska. Doctoral Dissertation. University of Belgrade. Republic of Serbia.

https://support.office.com/en-us/article/Load-the-Solver-Add-in-612926fc-d53b-46b4-872c-e24772f078ca

https://en.wikipedia.org/wiki/Operations_research